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Exclusive $c \to s, d$ semileptonic decays of ground-state spin-1/2 doubly charmed baryons

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ABSTRACT

We evaluate exclusive semileptonic decays of ground-state spin-1/2 doubly heavy charmed baryons driven by a $c \to s, d$ transition at the quark level. Our results for the form factors are consistent with heavy quark spin symmetry constraints which are valid in the limit of an infinitely massive charm quark and near zero recoil. Only a few exclusive semileptonic decay channels have been theoretically analyzed before. For those cases we find that our results are in a reasonable agreement with previous calculations.

1. Introduction

Doubly heavy baryons offer a unique opportunity to study QCD in the presence of heavy quarks as well as providing, through their decays, information on the weak sector of the Standard Model. From the experimental point of view the SELEX Collaboration claimed evidence for the Ξ_{cc}^+ baryon, in the $\Lambda_c^+ K^- \pi^+$ [1] and pD^+K^- [2] decay modes. The combined analysis gave a mass of $M_{\Xi_{cc}^+} = 3518.7 \pm 1.7\ \mathrm{MeV}/c^2$. However, other experimental collaborations like FOCUS [3], BaBar [4] and BELLE [5] found no evidence for doubly charmed baryons and the Ξ_{cc}^+ has only been assigned a one star status by the Particle Data Group (PDG) [6]. Furthermore, no evidence for the Ω_{cc}^+ has been reported so far. Nevertheless, being the lightest among the doubly heavy baryons, one expects doubly charmed baryons masses and decay properties to be measured in the near future.

While there are many different theoretical determinations of the doubly charmed baryon masses [7–28], that range from non-relativistic quark model calculations to unquenched lattice QCD, there are just a few studies of their decays.

Total decay widths were evaluated in Refs. [29–32], and total semileptonic and non-leptonic decay rates were predicted in Ref. [30]. Some exclusive non-leptonic as well as semileptonic decay rates of the Ξ_{cc} baryon were calculated in [31]. Finally the decay $\Xi_{cc} \to \Xi_c' e^+ \nu_e$ was analyzed in Ref. [33].[1] To our knowledge, there is not exist any systematic study of the exclusive semileptonic $c \to s$ and $c \to d$ decay channels of the Ξ_{cc} and Ω_{cc} baryons. This is the purpose of this work, where we shall concentrate in transitions to the lowest-lying, $1/2^+$ or $3/2^+$, single-c baryons in the final state. Besides, we will pay a special attention to possible violations of heavy quark spin symmetry relations among the relevant form factors, which one might expect to be sizable at the charm mass scale.

In Table 1, we show the quantum numbers of the baryons involved in our calculation. Quark model masses have been taken from our previous works in Refs. [25,34], where they were obtained using the AL1 potential of Refs. [35,36]. Experimental masses are the ones quoted by the PDG and in the table we quote the average over the different charge states. With the exception of the Ξ_{cc}, the agreement is fairly good. For the actual calculation of the decays we shall use experimental masses except for the Ξ_{cc}, which is not well established, and for the Ω_{cc} due to the absence of experimental data. In those two cases, we take our model predictions in Table 1 which are in agreement with different lattice estimates [13,17,26].

The Letter is organized as follows: In Section 2 we give general formulae for the semileptonic decay width and the form factor decomposition of the hadronic matrix elements of the weak current. In Section 3 we will find out heavy quark spin symmetry relations

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[1] Note that the Ξ_c' baryon here is denoted as Ξ_c in Ref. [33].

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Table 1
Quantum numbers of double-c and single-c heavy baryons involved in this study. J^π and I are the spin-parity and isospin of the baryon, while S^π is the spin-parity of the two heavy or the two light quark subsystem. n denotes a u or d quark.

Baryon	J^P	I	S^π	Quark content	Mass [MeV]	
					Quark model [25,34]	Experiment [6]
Ξ_{cc}	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	ccn	3613	3518.9
Ω_{cc}	$\frac{1}{2}^+$	0	1^+	ccs	3712	–
Λ_c	$\frac{1}{2}^+$	0	0^+	udc	2295	2286.5
Σ_c	$\frac{1}{2}^+$	1	1^+	nnc	2469	2453.6
Σ_c^*	$\frac{3}{2}^+$	1	1^+	nnc	2548	2518.0
Ξ_c	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	nsc	2474	2469.3
Ξ_c'	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	nsc	2578	2576.8
Ξ_c^*	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	nsc	2655	2645.9
Ω_c	$\frac{1}{2}^+$	0	1^+	ssc	2681	2695.2
Ω_c^*	$\frac{3}{2}^+$	0	1^+	ssc	2755	2765.9

between different form factors. Finally in Section 4 we present the results. The Letter contains also two appendices: In Appendix A we give a brief description of the baryon states within the model and the expressions for the wave functions of the different baryons and in Appendix B we relate the form factors to weak matrix elements and show how the latter ones are evaluated in the model.

2. Decay width and form factor decomposition of the hadronic current

The total decay width for semileptonic $c \to l$ transitions, with $l = s, d$, is given by

$$\Gamma = |V_{cl}|^2 \frac{G_F^2}{8\pi^4} \frac{M'^2}{M} \int \sqrt{w^2-1}\,\mathcal{L}^{\alpha\beta}(q)\mathcal{H}_{\alpha\beta}(P,P')\,dw \tag{1}$$

where $|V_{cl}|$ is the modulus of the corresponding Cabibbo–Kobayashi–Maskawa (CKM) matrix element for a $c \to l$ quark transition, for which we shall use $|V_{cs}| = 0.97345$ and $|V_{cd}| = 0.2252$ taken from Ref. [6]. $G_F = 1.16637(1) \times 10^{-11}$ MeV^{-2} [6] is the Fermi decay constant, P, M (P', M') are the four-momentum and mass of the initial (final) baryon, $q = P - P'$ and w is the product of the baryons four-velocities $w = v \cdot v' = \frac{P}{M} \cdot \frac{P'}{M'} = \frac{M^2+M'^2-q^2}{2MM'}$. In the decay, w ranges from $w = 1$, corresponding to zero recoil of the final baryon, to a maximum value given, neglecting the neutrino mass, by $w = w_{\max} = \frac{M^2+M'^2-m^2}{2MM'}$, which depends on the transition and where m is the final charged lepton mass. Finally $\mathcal{L}^{\alpha\beta}(q)$ is the leptonic tensor after integrating in the lepton momenta and $\mathcal{H}_{\alpha\beta}(P,P')$ is the hadronic tensor.

The leptonic tensor is given by

$$\mathcal{L}^{\alpha\beta}(q) = A(q^2)g^{\alpha\beta} + B(q^2)\frac{q^\alpha q^\beta}{q^2} \tag{2}$$

where

$$A(q^2) = -\frac{I(q^2)}{6}\left(2q^2 - m^2 - \frac{m^4}{q^2}\right), \qquad B(q^2) = \frac{I(q^2)}{3}\left(q^2 + m^2 - 2\frac{m^4}{q^2}\right) \tag{3}$$

with

$$I(q^2) = \frac{\pi}{2q^2}(q^2 - m^2) \tag{4}$$

The hadronic tensor reads

$$\mathcal{H}^{\alpha\beta}(P,P') = \frac{1}{2J+1}\sum_{r,r'}\langle B', r'\vec{P}'|J_{cl}^\alpha(0)|B, r\vec{P}\rangle\langle B', r'\vec{P}'|J_{cl}^\beta(0)|B, r\vec{P}\rangle^* \tag{5}$$

with J the initial baryon spin, $|B, r\vec{P}\rangle$ ($|B', r'\vec{P}'\rangle$) the initial (final) baryon state with three-momentum $\vec{P}$ ($\vec{P}'$) and spin third component r (r') in its center of mass frame. $J_{cl}^\mu(0)$ is the charged weak current for a $c \to l$ quark transition

$$J_{cl}^\mu(0) = \bar{\Psi}_l(0)\gamma^\mu(1-\gamma_5)\Psi_c(0) \tag{6}$$

Baryonic states are normalized such that

$$\langle B, r'\vec{P}'|B, r\vec{P}\rangle = 2E(2\pi)^3\delta_{rr'}\delta^3(\vec{P} - \vec{P}') \tag{7}$$

with E the baryon energy for three-momentum $\vec{P}$.

2.1. Form factors for $1/2 \to 1/2$ and $1/2 \to 3/2$ transitions

Hadronic matrix elements can be parameterized in terms of form factors. For $1/2 \to 1/2$ transitions the commonly used form factor decomposition reads

$$\begin{aligned}&\langle B'(1/2), r'\vec{P}'|\bar{\Psi}_l(0)\gamma^\mu(1-\gamma_5)\Psi_c(0)|B(1/2), r\vec{P}\rangle\\&\quad=\bar{u}^{B'}_{r'}(\vec{P}')\{\gamma^\mu[F_1(w)-\gamma_5 G_1(w)]+v^\mu[F_2(w)-\gamma_5 G_2(w)]+v'^\mu[F_3(w)-\gamma_5 G_3(w)]\}u^B_r(\vec{P})\end{aligned} \tag{8}$$

The u_r are Dirac spinors normalized as $(u_{r'})^\dagger u_r = 2E\delta_{rr'}$. v^μ, v'^μ are the four velocities of the initial and final baryons. The three vector F_1, F_2, F_3 and three axial G_1, G_2, G_3 form factors are functions of w or equivalently of q^2.

For $1/2 \to 3/2$ transitions we follow Llewellyn Smith [37] to write

$$\begin{aligned}&\langle B'(3/2), r'\vec{P}'|\bar{\Psi}_l(0)\gamma^\mu(1-\gamma_5)\Psi_c(0)|B(1/2), r\vec{P}\rangle=\bar{u}^{B'}_{\lambda r'}(\vec{P}')\Gamma^{\lambda\mu}(P,P')u^B_r(\vec{P})\\&\Gamma^{\lambda\mu}(P,P')=\left[\frac{C_3^V(w)}{M}(g^{\lambda\mu}\not{q}-q^\lambda\gamma^\mu)+\frac{C_4^V(w)}{M^2}(g^{\lambda\mu}qP'-q^\lambda P'^\mu)+\frac{C_5^V(w)}{M^2}(g^{\lambda\mu}qP-q^\lambda P^\mu)+C_6^V(w)g^{\lambda\mu}\right]\gamma_5\\&\qquad+\left[\frac{C_3^A(w)}{M}(g^{\lambda\mu}\not{q}-q^\lambda\gamma^\mu)+\frac{C_4^A(w)}{M^2}(g^{\lambda\mu}qP'-q^\lambda P'^\mu)+C_5^A(w)g^{\lambda\mu}+\frac{C_6^A(w)}{M^2}q^\lambda q^\mu\right]\end{aligned} \tag{9}$$

Here $u^{B'}_{\lambda r'}$ is the Rarita–Schwinger spinor of the final spin 3/2-baryon normalized such that $(u^{B'}_{\lambda r'})^\dagger u^{B'\lambda}_r = -2E'\delta_{rr'}$, and we have four vector $(C^V_{3,4,5,6}(w))$ and four axial $(C^A_{3,4,5,6}(w))$ form factors.

In Appendix B we give the expressions that relate the form factors to weak current matrix elements and show how the latter ones are evaluated within the model.

3. Heavy quark spin symmetry

In hadrons with a single heavy quark the dynamics of the light degrees of freedom becomes independent of the heavy quark flavour and spin when the mass of the heavy quark is much larger than Λ_{QCD} and the masses and momenta of the light quarks. This is the essence of heavy quark symmetry (HQS) [38–41]. However, HQS cannot be directly applied to hadrons containing two heavy quarks. The static theory for a system with two heavy quarks has infra-red divergences which can be regulated by the kinetic energy term $\bar{h}_Q(D^2/2m_Q)h_Q$. This term breaks the heavy quark flavour symmetry, but not the spin symmetry for each heavy quark flavour [42]. This is known as heavy quark spin symmetry (HQSS). HQSS implies that all baryons listed in Table 1 with the same flavour wave function are degenerate. The invariance of the effective Lagrangian under arbitrary spin rotations of the c quark leads to relations, near the zero recoil point ($w = 1 \leftrightarrow q^2 = (M-M')^2 \leftrightarrow |\vec{q}| = 0$), between the form factors for vector and axial-vector currents between the Ξ_{cc} and Ω_{cc} baryons and the single charmed baryons listed in Table 1. These decays are induced by the semileptonic weak decay of the c quark to a d or a s quark. The consequences of spin symmetry for weak matrix elements can be derived using the "trace formalism" [43,44]. To represent the lowest-lying S-wave ccl baryons we will use wave functions comprising tensor products of Dirac matrices and spinors, namely [45][2]:

$$\Xi_{cc} = -\sqrt{\frac{1}{3}}\left[\frac{(1+\not{v})}{2}\gamma_5\right]_{\alpha\beta} u_\gamma(v,r) \tag{10}$$

where we have indicated Dirac indices α, β and γ explicitly on the right-hand side and r is a helicity label for the baryon. Under a Lorentz transformation, Λ, and a c quark spin transformation S_c, this wave function of the form $\Gamma_{\alpha\beta}u_\gamma$ transforms as:

$$\Gamma u \to S(\Lambda)\Gamma S^{-1}(\Lambda)\,S(\Lambda)u, \qquad \Gamma u \to S_c\Gamma S_c u \tag{11}$$

The state in Eq. (10) is normalized[3] to $(-\bar{u}u = -2M)$, with M the mass of the state. On the other hand, the Λ_c, Σ_c and Σ_c^* final baryons are represented by the following spinor wave functions [44]

$$\Lambda_c = u_\gamma(v',r') \tag{12}$$

$$\Sigma_c = \left[\frac{1}{\sqrt{3}}(v'^\lambda+\gamma^\lambda)\gamma_5 u(v',r')\right]_\gamma \tag{13}$$

$$\Sigma_c^* = u^\lambda_\gamma(v',r') \tag{14}$$

For the Σ_c^*, $u^\lambda_\gamma(v',r')$ is a Rarita–Schwinger spinor. For Σ_c, we have taken into account that the light quarks are coupled to total spin 1 that gives a total spin 1/2 for the baryon when the spin of the light subsystem is summed with the spin of the charm quark. Under a Lorentz transformation, Λ, and a c quark spin transformation S_c, the above spinor wave functions transform like $S(\Lambda)\mathcal{U}$ and $S_c\mathcal{U}$, respectively, with $\mathcal{U}$ $(= u, \frac{1}{\sqrt{3}}(v'^\lambda+\gamma^\lambda)\gamma_5 u, u^\lambda)$ each of the spinors appearing in Eqs. (12)–(14). States are normalized to $\bar{u}u = 2M'$ $(-\bar{u}u = -2M')$ and $\bar{u}_\lambda u^\lambda = -2M'$ for the Λ_c, Σ_c and Σ_c^*, respectively.

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[2] We will give here expressions only for the $c \to d$ transitions of the Ξ_{cc} baryon. Expressions for the Ω_{cc} initial baryon and/or $c \to s$ transitions are totally similar, and SU(3) flavour symmetry could be used to establish relations between the former and the latter ones.

[3] Note, there are two ways to contract the charm quark indices, leading to $\bar{u}u\,\mathrm{Tr}(\Gamma\bar{\Gamma}) + \bar{u}\Gamma\bar{\Gamma}u$, with $\bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$.

We can now construct amplitudes for semileptonic $\Xi_{cc} \to \Lambda_c, \Sigma_c, \Sigma_c^*$ decays, determined by matrix elements of the weak current $J^\mu = \bar{d}\gamma^\mu(1-\gamma_5)c$. To that end, we write the most general form for the matrix element respecting the heavy quark spin symmetry, taking into account that under a c quark spin transformation $J^\mu \to J^\mu S_c^\dagger$. We should distinguish two situations depending on whether the total spin of the two light quarks in the final baryon is $S=0$ or $S=1$. In the first (second) case, the spinor wave function $\mathcal{U}$ that represents the final baryon does not have (has) a Lorentz index. With all these considerations, we have

$$\begin{aligned}
&\langle \Lambda_c, v', r' | J^\mu(0) | \Xi_{cc}, v, r\rangle \\
&\quad = \bar{u}_{\Lambda_c}(v', r') \frac{(1+\not{v})}{2}\gamma_5 \Omega \gamma^\mu (1-\gamma_5) u_{\Xi_{cc}}(v, r) + \bar{u}_{\Lambda_c}(v', r') u_{\Xi_{cc}}(v, r)\, \mathrm{Tr}\left[\frac{(1+\not{v})}{2}\gamma_5 \Omega \gamma^\mu (1-\gamma_5)\right] \qquad (15)\\
&\langle \Sigma_c, v', r' | J^\mu(0) | \Xi_{cc}, v, r\rangle \\
&\quad = \bar{u}_{\Sigma_c}(v', r') \frac{1}{\sqrt{3}}(\gamma^\lambda - v'^\lambda)\gamma_5 \frac{(1+\not{v})}{2}\gamma_5 \Omega_\lambda \gamma^\mu (1-\gamma_5) u_{\Xi_{cc}}(v, r) \\
&\qquad + \bar{u}_{\Sigma_c}(v', r') \frac{1}{\sqrt{3}}(\gamma^\lambda - v'^\lambda)\gamma_5 u_{\Xi_{cc}}(v, r)\, \mathrm{Tr}\left[\frac{(1+\not{v})}{2}\gamma_5 \Omega_\lambda \gamma^\mu (1-\gamma_5)\right] \qquad (16)\\
&\langle \Sigma_c^*, v', r' | J^\mu(0) | \Xi_{cc}, v, r\rangle \\
&\quad = \bar{u}^\lambda_{\Sigma_c^*}(v', r') \frac{(1+\not{v})}{2}\gamma_5 \Omega_\lambda \gamma^\mu (1-\gamma_5) u_{\Xi_{cc}}(v, r) + \bar{u}^\lambda_{\Sigma_c^*}(v', r') u_{\Xi_{cc}}(v, r)\, \mathrm{Tr}\left[\frac{(1+\not{v})}{2}\gamma_5 \Omega_\lambda \gamma^\mu (1-\gamma_5)\right] \qquad (17)
\end{aligned}$$

with[4]

$$\Omega = \beta_1(w) + \beta_2(w)\not{v}' \qquad (18)$$

$$\Omega_\lambda = \delta_1(w)v_\lambda + \delta_2(w)\gamma_\lambda + \delta_3(w)\not{v}'v_\lambda + \delta_4\not{v}'\gamma_\lambda \qquad (19)$$

Note that near the zero recoil point, where the spin symmetry should work best, HQSS considerably reduces the number of independent form factors, and it relates those that correspond to transitions where the spin of the two light quarks in the final baryon is $S=1$. Indeed, we find at $w=1$

- $1/2 \to 1/2$ transitions ($\Xi_{cc} \to \Lambda_c, \Xi_c$ and $\Omega_{cc} \to \Xi_c$), where the total spin of the two light quarks in the final baryon is $S=0$:

  $$F_1 + F_2 + F_3 = 3G_1 \equiv \eta_0 \qquad (20)$$

  In the equal mass transition case one would find that η_0 is normalized as $\eta_0(w=1) = \sqrt{\frac{3}{2}}$.
- Total spin of the two light quarks in the final baryon is $S=1$.
  - $1/2 \to 1/2$ transitions ($\Xi_{cc} \to \Sigma_c, \Xi'_c$ and $\Omega_{cc} \to \Xi'_c, \Omega_c$).

    $$F_1 + F_2 + F_3 = \frac{3}{5}G_1 \equiv \eta_1 \qquad (21)$$

  - $1/2 \to 3/2$ transitions ($\Xi_{cc} \to \Sigma_c^*, \Xi_c^*$ and $\Omega_{cc} \to \Xi_c^*, \Omega_c^*$).

    $$\frac{\sqrt{3}}{2}\left(C_3^A \frac{M-M'}{M} + C_4^A \frac{M'(M-M')}{M^2} + C_5^A\right) = \eta_1 \qquad (22)$$

  In the equal mass transition case one would have that $\eta_1(w=1) = \frac{1}{\sqrt{2}}$ when the two light quarks in the final state are different and $\eta_1(w=1) = 1$ when they are equal (Ω_c and Ω_c^*).

Relations (20), (21) and (22) are exactly satisfied in the quark model when the heavy quark mass is made arbitrarily large, and thus the calculation is consistent with HQSS constraints.

4. Results and discussion

We start by checking that our calculation respects the constraints on the form factors deduced from HQSS. In Figs. 1 and 2, we show to what extent the relations of (20), (21) and (22) deduced above are satisfied for the actual m_c value. In all cases we see moderate deviations, that stem from $1/m_c$ corrections, at the level of about 10% near zero recoil, though larger than those found in [46] for the $b \to c$ transitions of the Ξ_{bc} and Ξ_{bb} baryons. These discrepancies tend to disappear when the mass of the heavy quark is made arbitrarily large. This is illustrated in Fig. 3 where we show, for $w=1$ and for three different heavy quark masses, the form factor ratio $\frac{3G_1}{F_1+F_2+F_3}$ from the $\Xi_{cc}^{++} \to \Xi_c^+$ transition, the form factor ratio $\frac{3/5G_1}{F_1+F_2+F_3}$ for the $\Omega_{cc}^+ \to \Omega_c^0$ transition and the ratio $\frac{\sqrt{3}}{2}(C_3^A\frac{M-M'}{M} + C_4^A\frac{M'(M-M')}{M^2} + C_5^A)/(F_1+F_2+F_3)$ constructed with the $C_{3,4,5}^A$ form factors from the $\Omega_{cc}^+ \to \Omega_c^{*0}$ transition and the $F_{1,2,3}$ from the $\Omega_{cc}^+ \to \Omega_c^0$ one. The ratios are shown as a function of the corresponding pseudoscalar P heavy–light meson mass. As the pseudoscalar meson mass increases (the heavy quark mass

[4] Terms with a factor of $\not{v}$ can be omitted because $\not{v}(1 \pm \not{v}) = \pm(1 \pm \not{v})$.

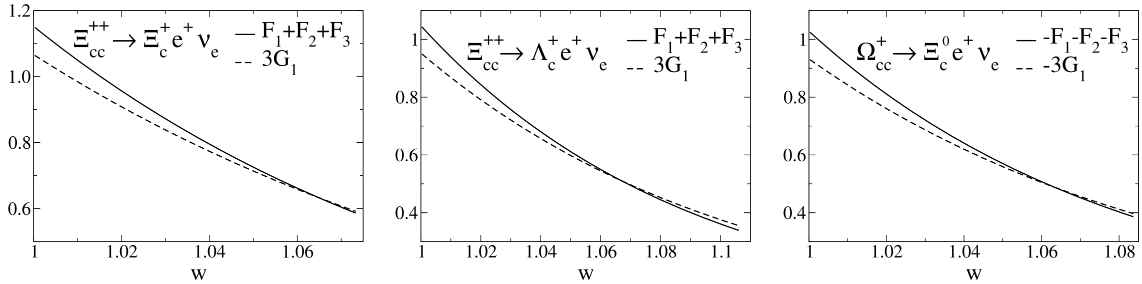

Fig. 1. Comparison of $F_1+F_2+F_3$ (solid) and $3G_1$ (dashed) for the specified transitions. The two light quarks in the final baryon have total spin $S=0$. In the limit in which the heavy quark mass is made arbitrarily large one has that, near zero recoil ($w=1$), $F_1+F_2+F_3=3G_1$.

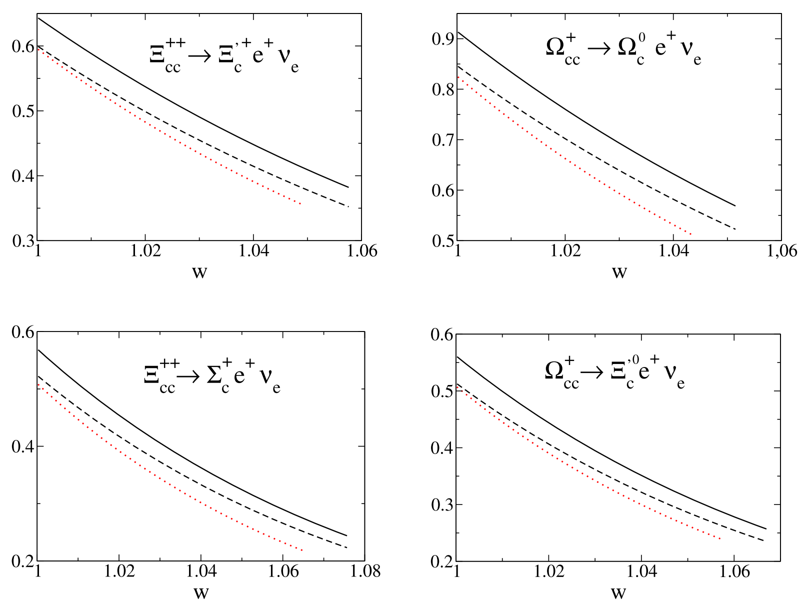

Fig. 2. Solid (dashed): $F_1+F_2+F_3$ ($3G_1/5$) for the specified transitions. Dotted: the combination $\frac{\sqrt{3}}{2}(C_3^A\frac{M-M'}{M}+C_4^A\frac{M'(M-M')}{M^2}+C_5^A)$ for the transition with the corresponding 3/2 baryon (Σ_c^*, Ξ_c^* or Ω_c^*) in the final state. In all cases the two light quarks in the final baryon have total spin $S=1$. In the limit in which the heavy quark mass is made arbitrarily large one has that, near zero recoil ($w=1$), $F_1+F_2+F_3=\frac{3}{5}G_1=\frac{\sqrt{3}}{2}(C_3^A\frac{M-M'}{M}+C_4^A\frac{M'(M-M')}{M^2}+C_5^A)$.

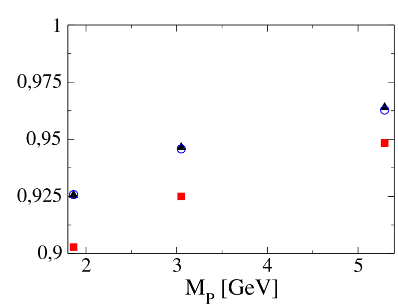

Fig. 3. Form factor ratio $\frac{3G_1}{F_1+F_2+F_3}$ (open circles) from the $\Xi_{cc}^{++}\to\Xi_c^+$ transition, form factor ratio $\frac{3/5G_1}{F_1+F_2+F_3}$ (up triangles) for the $\Omega_{cc}^+\to\Omega_c^0$ transition and the ratio $\frac{\frac{\sqrt{3}}{2}(C_3^A\frac{M-M'}{M}+C_4^A\frac{M'(M-M')}{M^2}+C_5^A)}{F_1+F_2+F_3}$ (squares) constructed with the $C_{3,4,5}^A$ form factors from the $\Omega_{cc}^+\to\Omega_c^{*0}$ transition and the $F_{1,2,3}$ from the $\Omega_{cc}^+\to\Omega_c^0$ one. Ratios are shown as a function of the pseudoscalar P heavy–light meson mass for three different heavy quark masses and for $w=1$.

increases) the ratios tend to one as expected. Similar results are obtained in the other cases. Even though we are not in the infinite heavy quark mass limit, HQSS turns out to be a useful tool to understand the dynamics of the $c\to s,d$ Ξ_{cc} and Ω_{cc} decays near zero recoil. One also sees that at $w=1$ our results for $\eta_0(w=1)$, $\eta_1(w=1)$ are systematically smaller than would be expected for an equal mass transition. This is a reflection of the mismatch in the wave functions due to the different initial (c) and final (d or s) quark masses in the $c\to d,s$ decays.

Now we discuss the results for the decay widths. Those are shown in Table 2 for the dominant ($c\to s$) and sub-dominant ($c\to d$) exclusive semileptonic decays of the Ξ_{cc} and Ω_{cc} to ground state, $1/2^+$ or $3/2^+$, single charmed baryons and with a positron in the final

Table 2
Decay widths in units of ps^{-1}. We use $|V_{cs}| = 0.97345$ and $|V_{cd}| = 0.2252$ taken from Ref. [6]. Results with an $(*)$, our estimates from the total decay widths and branching ratios in [31]. Similar results are obtained for $\mu^+\nu_\mu$ leptons in the final state.

$B_{cc} \to B_c e^+ \nu_e$	Quark transition	Γ [ps^{-1}]		
		This work	[33]	[31]
$\Xi_{cc}^{++} \to \Xi_c^{+} e^+ \nu_e$	$(c \to s)$	8.75×10^{-2}		
$\Xi_{cc}^{+} \to \Xi_c^{0} e^+ \nu_e$	$(c \to s)$	8.68×10^{-2}		
$\Xi_{cc}^{++} \to \Xi_c^{\prime +} e^+ \nu_e$	$(c \to s)$	0.146	0.208–0.258	
$\Xi_{cc}^{+} \to \Xi_c^{\prime 0} e^+ \nu_e$	$(c \to s)$	0.145	0.208–0.258	
$\Xi_{cc}^{++} \to \Xi_c^{*+} e^+ \nu_e$	$(c \to s)$	3.20×10^{-2}		
$\Xi_{cc}^{+} \to \Xi_c^{*0} e^+ \nu_e$	$(c \to s)$	3.20×10^{-2}		
$\Xi_{cc}^{++} \to \Xi_c^{\prime +} e^+ \nu_e + \Xi_c^{+} e^+ \nu_e + \Xi_c^{*+} e^+ \nu_e$	$(c \to s)$	0.266		$0.37 \pm 0.04^{(*)}$
$\Xi_{cc}^{+} \to \Xi_c^{\prime 0} e^+ \nu_e + \Xi_c^{0} e^+ \nu_e + \Xi_c^{*0} e^+ \nu_e$	$(c \to s)$	0.264		$0.47 \pm 0.15^{(*)}$
$\Xi_{cc}^{++} \to \Lambda_c^{+} e^+ \nu_e$	$(c \to d)$	4.86×10^{-3}		
$\Xi_{cc}^{++} \to \Sigma_c^{+} e^+ \nu_e$	$(c \to d)$	7.94×10^{-3}		
$\Xi_{cc}^{+} \to \Sigma_c^{0} e^+ \nu_e$	$(c \to d)$	1.58×10^{-2}		
$\Xi_{cc}^{++} \to \Sigma_c^{*+} e^+ \nu_e$	$(c \to d)$	1.77×10^{-3}		
$\Xi_{cc}^{+} \to \Sigma_c^{*0} e^+ \nu_e$	$(c \to d)$	3.54×10^{-3}		
$\Omega_{cc}^{+} \to \Omega_c^{0} e^+ \nu_e$	$(c \to s)$	0.282		
$\Omega_{cc}^{+} \to \Omega_c^{*0} e^+ \nu_e$	$(c \to s)$	5.77×10^{-2}		
$\Omega_{cc}^{+} \to \Xi_c^{0} e^+ \nu_e$	$(c \to d)$	4.11×10^{-3}		
$\Omega_{cc}^{+} \to \Xi_c^{\prime 0} e^+ \nu_e$	$(c \to d)$	7.44×10^{-3}		
$\Omega_{cc}^{+} \to \Xi_c^{*0} e^+ \nu_e$	$(c \to d)$	1.72×10^{-3}		

state.[5] For the Ω_{cc}^{+} baryon, semileptonic decays driven by a $s \to u$ transition at the quark level are also possible. However, in this latter case phase space is very limited and we find the decay widths are orders of magnitude smaller than the ones shown. To our knowledge there are just a few previous theoretical evaluations of the Ξ_{cc} semileptonic decays. In Ref. [33] the authors use the relativistic three-quark model to evaluate the $\Xi_{cc} \to \Xi_c^{\prime} e^+ \nu_e$ decay, while in Ref. [31], using heavy quark effective theory and non-relativistic QCD sum rules, they give both the lifetime of the Ξ_{cc} baryon and the branching ratio for the combined decay $\Xi_{cc} \to \Xi_c e^+ \nu_e + \Xi_c^{\prime} e^+ \nu_e + \Xi_c^{*} e^+ \nu_e$ from which we have evaluated the semileptonic decay widths shown in the table. We find a fair agreement of our predictions with both calculations. In Ref. [30], using the optical theorem and the operator product expansion, the authors evaluated the total semileptonic decay rate finding it to be 0.151 ps^{-1} for Ξ_{cc}^{++} and 0.166 ps^{-1} for Ξ_{cc}^{+}. These values are roughly a factor of two smaller than the sum of our partial decay widths or the results in Ref. [31]. For the Ω_{cc}^{+} a total semileptonic decay width of 0.454 ps^{-1} is given in Ref. [30]. In this case this is in better agreement with the sum of our partial semileptonic decay widths which add up to 0.353 ps^{-1}.

An estimate of part of the uncertainties in our model can be done by evaluating the decay widths using wave functions produced with different interquark interactions. We have done this by using the AP1 [35,36] and Bhaduri [47] interquark potentials finding changes in the decay widths to be at the level of 10%. Another source of uncertainties may come from the contribution from intermediate heavy–light vector meson (D^* and D_s^*) exchanges [48]. They are neither considered in this work nor in the previous quark model calculation of Ref. [33].[6] We expect such exchanges to produce small effects[7] in the integrated widths, specially for the decays considered in this work, for which the D^* and D_s^* poles are located far from $\sqrt{q^2_{\text{max}}}$. This is in sharp contrast with the situation for the $B \to \pi$ and $D \to \pi$ decays [48,49]. The model could be also improved by considering two body operators, and going in this manner beyond the spectator approximation. However, two body current contributions are not straightforward to compute, and since we expect moderate effects,[8] similar to the other uncertainties mentioned above, we will leave this issue for future research. Moreover, there exists a greater source of uncertainties affecting our results that comes from our limited knowledge on the masses of the initial double charmed baryons. As we pointed out in the introduction, for the Ξ_{cc} and the Ω_{cc} baryons, we have used our quark model predictions in Table 1. If the SELEX Collaboration measured mass for the Ξ_{cc} baryon is used instead, we would find significantly smaller decay widths by about 20%. This is just because of the reduction on the available phase-space for the decay. None of the theoretical works mentioned in Table 2 use the SELEX mass value.

To summarize this work, we would like to point out that we have carried out the first systematic study of all dominant and sub-dominant semileptonic transitions of the doubly charmed Ξ_{cc} and Ω_{cc} baryons to the lowest-lying, $1/2^+$ or $3/2^+$, single-c baryons. To that end, we have employed a simple constituent quark model scheme, which benefits from the important simplifications [21,34] of the non-relativistic three body problem that stem from the application of HQSS. We have also derived, for the first time, HQSS relations among the relevant form factors that govern these decays near zero recoil, and have found the size of the deviations induced by the finite charm quark mass.

Predictions of this framework have been successfully tested in the past in the context of the Λ_b and Ξ_b semileptonic decays [50]. There, we obtained results for partially integrated decay widths that nicely compared with lattice results [51], and from the experimental Λ_b-semileptonic decay, we could also determine the V_{cb} CKM matrix element in excellent agreement with the accepted values quoted in the PDG [6].

5 Similar results are obtained for $\mu^+\nu_\mu$ leptons in the final state.

6 We think, these effects are not explicitly taken into account either in the QCD sum rule approach of Ref. [31] or in that, based in the optical theorem, followed in [30].

7 Moreover in the transitions studied here, the intermediate vector mesons would be far off shell. Thus, the uncertainties related to the strength of their couplings with the singly and doubly charmed baryons, and those stemming from the lack of a reasonable scheme to model how the latter interactions are suppressed when q^2 approaches the endpoint of the available phase-space ($q^2 = 0$) would make meaningless the computation of these effects.

8 The difference between the sum of masses of the constituent quarks and that of the baryon provides a first estimate of these effects [50].

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Appendix A. Non-relativistic baryon states and wave functions

Our non-relativistic states are constructed as a superposition of three quark states

$$|B, r\vec{P}\rangle_{NR} = \sqrt{2E}\int d^3Q_1 \int d^3Q_2 \frac{1}{\sqrt{2}} \sum_{\alpha_1,\alpha_2,\alpha_3} \hat{\psi}^{(B,r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2)\frac{1}{(2\pi)^3\sqrt{2E_{f_1}2E_{f_2}2E_{f_3}}} \times \left|\alpha_1\vec{p}_1 = \frac{m_{f_1}}{\bar{M}}\vec{P}+\vec{Q}_1\right\rangle\left|\alpha_2\vec{p}_2 = \frac{m_{f_2}}{\bar{M}}\vec{P}+\vec{Q}_2\right\rangle\left|\alpha_3\vec{p}_3 = \frac{m_{f_3}}{\bar{M}}\vec{P}-\vec{Q}_1-\vec{Q}_2\right\rangle \quad (A.1)$$

The factor $\sqrt{2E}$ is introduced for convenience in order to have the proper normalization. α_j represents the spin (s), flavour (f) and color (c) quantum numbers ($\alpha \equiv (s, f, c)$) of the j-th quark, and $(E_{f_j}, \vec{p}_j)$, m_{f_j} are its four-momentum and mass. $\bar{M}$ is given by $\bar{M} = m_{f_1}+m_{f_2}+m_{f_3}$. Individual quark states are normalized such that $\langle\alpha'\vec{p}'|\alpha\vec{p}\rangle = 2E_f(2\pi)^3\delta_{\alpha'\alpha}\delta^3(\vec{p}'-\vec{p})$. $\hat{\psi}^{(B,r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2)$ is the internal wave function in momentum space, being $\vec{Q}_1$ ($\vec{Q}_2$) the conjugate momenta to the relative position $\vec{r}_1$ ($\vec{r}_2$) between the third quark and quark 1 (2). In the transitions under study an initial ccl' baryon decays into a final cll' one, where $l = d, s$ and $l' = u, d, s$. We construct the wave functions such that the two c quarks in the initial baryon, or the two light quarks in the final baryon, are quarks 1 and 2. Expressions for the different $\hat{\psi}^{(B,r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2)$ are given below. These wave functions are normalized as

$$\int d^3Q_1\int d^3Q_2 \sum_{\alpha_1,\alpha_2,\alpha_3}\left(\hat{\psi}^{(B,r')}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2)\right)^*\hat{\psi}^{(B,r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2) = \delta_{rr'} \quad (A.2)$$

so that we get for our non-relativistic baryon states ${}_{NR}\langle B, r'\vec{P}'|B, r\vec{P}\rangle_{NR} = 2E(2\pi)^3\delta_{rr'}\delta^3(\vec{P}'-\vec{P})$.

The wave functions of the different non-strange states included in this study are given by

$$\hat{\psi}^{(\Xi_{cc}^{++},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2) = \frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\tilde{\phi}^{(\Xi_{cc}^{++})}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1c}\delta_{f_2c}\delta_{f_3u}(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,1/2;s_1+s_2,s_3,r) \quad (A.3)$$

$$\hat{\psi}^{(\Xi_{cc}^{+},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2) = \frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\tilde{\phi}^{(\Xi_{cc}^{+})}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1c}\delta_{f_2c}\delta_{f_3d}(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,1/2;s_1+s_2,s_3,r) \quad (A.4)$$

$$\hat{\psi}^{(\Lambda_{c}^{+},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2) = \frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\tilde{\phi}^{(\Lambda_{c}^{+})}(\vec{Q}_1,\vec{Q}_2)\frac{1}{\sqrt{2}}(\delta_{f_1u}\delta_{f_2d}-\delta_{f_1d}\delta_{f_2u})\delta_{f_3c}(1/2,1/2,0;s_1,s_2,0)\delta_{s_3r} \quad (A.5)$$

$$\hat{\psi}^{(\Sigma_{c}^{+},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2) = \frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\tilde{\phi}^{(\Sigma_{c}^{+})}(\vec{Q}_1,\vec{Q}_2)\frac{1}{\sqrt{2}}(\delta_{f_1u}\delta_{f_2d}+\delta_{f_1d}\delta_{f_2u})\delta_{f_3c}(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,1/2;s_1+s_2,s_3,r) \quad (A.6)$$

$$\hat{\psi}^{(\Sigma_{c}^{0},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2) = \frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\tilde{\phi}^{(\Sigma_{c}^{0})}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1d}\delta_{f_2d}\delta_{f_3c}(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,1/2;s_1+s_2,s_3,r) \quad (A.7)$$

$$\hat{\psi}^{(\Sigma_{c}^{*+},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2) = \frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\tilde{\phi}^{(\Sigma_{c}^{*+})}(\vec{Q}_1,\vec{Q}_2)\frac{1}{\sqrt{2}}(\delta_{f_1u}\delta_{f_2d}+\delta_{f_1d}\delta_{f_2u})\delta_{f_3c}(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,3/2;s_1+s_2,s_3,r) \quad (A.8)$$

$$\hat{\psi}^{(\Sigma_{c}^{*0},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2) = \frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\tilde{\phi}^{(\Sigma_{c}^{*0})}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1d}\delta_{f_2d}\delta_{f_3c}(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,3/2;s_1+s_2,s_3,r) \quad (A.9)$$

where $\varepsilon_{c_1c_2c_3}$ is the totally antisymmetric tensor with $\frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}$ being the fully antisymmetric color wave function. The $(j_1, j_2, j; m_1, m_2, m)$ are SU(2) Clebsch–Gordan coefficients. The different $\tilde{\phi}(\vec{Q}_1,\vec{Q}_2)$ wave functions verify $\tilde{\phi}(\vec{Q}_2,\vec{Q}_1) = \tilde{\phi}(\vec{Q}_1,\vec{Q}_2)$ and they have total orbital angular momentum 0 being invariant under rotations and thus depending only on $|\vec{Q}_1|$, $|\vec{Q}_2|$ and $\vec{Q}_1\cdot\vec{Q}_2$. They are normalized such that

$$\int d^3Q_1\int d^3Q_2\left|\tilde{\phi}(\vec{Q}_1,\vec{Q}_2)\right|^2 = 1 \quad (A.10)$$

For states with s-quark content we further have

$$\hat{\psi}^{(\Omega_{cc}^{+},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2) = \frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\tilde{\phi}^{(\Omega_{cc}^{+})}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1c}\delta_{f_2c}\delta_{f_3s}(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,1/2;s_1+s_2,s_3,r) \quad (A.11)$$

$$\hat{\psi}^{(\Xi_{c}^{+},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2) = \frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\frac{1}{\sqrt{2}}\left[\tilde{\phi}^{(\Xi_{c}^{+})}_{us}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1u}\delta_{f_2s}-\tilde{\phi}^{(\Xi_{c}^{+})}_{su}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1s}\delta_{f_2u}\right]\delta_{f_3c}(1/2,1/2,0;s_1,s_2,0)\delta_{s_3r} \quad (A.12)$$

$$\hat{\psi}^{(\Xi_{c}^{0},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2) = \frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\frac{1}{\sqrt{2}}\left[\tilde{\phi}^{(\Xi_{c}^{0})}_{ds}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1d}\delta_{f_2s}-\tilde{\phi}^{(\Xi_{c}^{0})}_{sd}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1s}\delta_{f_2d}\right]\delta_{f_3c}(1/2,1/2,0;s_1,s_2,0)\delta_{s_3r} \quad (A.13)$$

$$\hat{\psi}^{(\Xi_c'^{+},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2)=\frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\frac{1}{\sqrt{2}}\big[\tilde{\phi}^{(\Xi_c'^{+})}_{us}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1 u}\delta_{f_2 s}+\tilde{\phi}^{(\Xi_c'^{+})}_{su}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1 s}\delta_{f_2 u}\big]\delta_{f_3 c}\times(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,1/2;s_1+s_2,s_3,r) \tag{A.14}$$

$$\hat{\psi}^{(\Xi_c'^{0},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2)=\frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\frac{1}{\sqrt{2}}\big[\tilde{\phi}^{(\Xi_c'^{0})}_{ds}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1 d}\delta_{f_2 s}+\tilde{\phi}^{(\Xi_c'^{0})}_{sd}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1 s}\delta_{f_2 d}\big]\delta_{f_3 c}\times(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,1/2;s_1+s_2,s_3,r) \tag{A.15}$$

$$\hat{\psi}^{(\Xi_c^{*+},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2)=\frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\frac{1}{\sqrt{2}}\big[\tilde{\phi}^{(\Xi_c^{*+})}_{us}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1 u}\delta_{f_2 s}+\tilde{\phi}^{(\Xi_c^{*+})}_{su}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1 s}\delta_{f_2 u}\big]\delta_{f_3 c}\times(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,3/2;s_1+s_2,s_3,r) \tag{A.16}$$

$$\hat{\psi}^{(\Xi_c^{*0},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2)=\frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\frac{1}{\sqrt{2}}\big[\tilde{\phi}^{(\Xi_c^{*0})}_{ds}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1 d}\delta_{f_2 s}+\tilde{\phi}^{(\Xi_c^{*0})}_{sd}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1 s}\delta_{f_2 d}\big]\delta_{f_3 c}\times(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,3/2;s_1+s_2,s_3,r) \tag{A.17}$$

$$\hat{\psi}^{(\Omega_c^{0},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2)=\frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\tilde{\phi}^{(\Omega_c^{0})}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1 s}\delta_{f_2 s}\delta_{f_3 c}(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,1/2;s_1+s_2,s_3,r) \tag{A.18}$$

$$\hat{\psi}^{(\Omega_c^{*0},r)}_{\alpha_1\alpha_2\alpha_3}(\vec{Q}_1,\vec{Q}_2)=\frac{\varepsilon_{c_1c_2c_3}}{\sqrt{3!}}\tilde{\phi}^{(\Omega_c^{*0})}(\vec{Q}_1,\vec{Q}_2)\delta_{f_1 s}\delta_{f_2 s}\delta_{f_3 c}(1/2,1/2,1;s_1,s_2,s_1+s_2)(1,1/2,1/2;s_1+s_2,s_3,r) \tag{A.19}$$

Here, besides the properties above, the relation $\tilde{\phi}_{sn}(\vec{Q}_1,\vec{Q}_2)=\tilde{\phi}_{ns}(\vec{Q}_2,\vec{Q}_1)$, with $n=u,d$, also applies.

These momentum space wave functions are the Fourier transform of the corresponding wave functions in coordinate space. Details on how the latter are evaluated in our model for singly and doubly heavy baryons can be found in Refs. [21,34].

The two baryons states Ξ_c, Ξ_c' differ just in the spin of the light degrees of freedom, and thus they could mix under the effect of the hyperfine interaction between the c quark and any of the light quarks. We have evaluated this mixing in our model finding it negligible.[9] Using the AL1 potential, the physical states resulting from the mixing are $\Xi_c^{(1)}=0.999\Xi_c-0.0437\Xi_c'$ and $\Xi_c^{(2)}=0.0437\Xi_c+0.999\Xi_c'$, being the mass changes of just 0.2 MeV with respect to the unmixed state case. We neglect this small mixing in our calculation.

Appendix B. Form factors and weak matrix elements

Taking the initial baryon at rest and $\vec{q}$ in the positive Z direction we define vector and axial matrix elements

$$V^{\mu}_{r\to r'}-A^{\mu}_{r\to r'}=\big\langle B',r'\vec{P}'=-\vec{q}\big|\bar{\Psi}_l(0)\gamma^{\mu}(1-\gamma_5)\Psi_c(0)\big|B,r\vec{P}=\vec{0}\big\rangle \tag{B.1}$$

In terms of matrix elements, the different form factors for the spin 1/2-baryon to spin 1/2-baryon transitions can be evaluated as

$$F_1=-\sqrt{\frac{E'+M'}{2M}}\frac{1}{|\vec{q}|}V^1_{-1/2\to1/2} \tag{B.2}$$

$$F_2=\frac{1}{\sqrt{(E'+M')2M}}\left(V^0_{1/2\to1/2}+\frac{E'}{|\vec{q}|}V^3_{1/2\to1/2}+\frac{M'}{|\vec{q}|}V^1_{-1/2\to1/2}\right) \tag{B.3}$$

$$F_3=-\frac{1}{\sqrt{(E'+M')2M}}\frac{M'}{|\vec{q}|}\big(V^3_{1/2\to1/2}-V^1_{-1/2\to1/2}\big) \tag{B.4}$$

$$G_1=\frac{1}{\sqrt{(E'+M')2M}}A^1_{-1/2\to1/2} \tag{B.5}$$

$$G_2=\sqrt{\frac{E'+M'}{2M}}\frac{1}{|\vec{q}|}\left(A^0_{1/2\to1/2}-\frac{M'}{|\vec{q}|}A^1_{-1/2\to1/2}+\frac{E'}{|\vec{q}|}A^3_{1/2\to1/2}\right) \tag{B.6}$$

$$G_3=-\sqrt{\frac{E'+M'}{2M}}\frac{M'}{|\vec{q}|^2}\big(A^3_{1/2\to1/2}-A^1_{-1/2\to1/2}\big) \tag{B.7}$$

For the spin 1/2-baryon to spin 3/2-baryon case the relations between form factors and weak matrix elements are

$$C_3^V=\frac{M'}{|\vec{q}|}\frac{1}{\sqrt{(E'+M')2M}}\frac{1}{\sqrt{2}}\big(V^1_{1/2\to3/2}+\sqrt{3}V^1_{1/2\to-1/2}\big) \tag{B.8}$$

$$C_4^V=\frac{1}{|\vec{q}|^3}\sqrt{\frac{E'+M'}{2M}}\frac{1}{\sqrt{2}}\big(-\sqrt{3}MM'V^3_{1/2\to1/2}+M\big(-2E'+M'\big)V^1_{1/2\to3/2}+\sqrt{3}MM'V^1_{1/2\to-1/2}\big) \tag{B.9}$$

$$C_5^V=\frac{1}{|\vec{q}|^3}\sqrt{\frac{E'+M'}{2M}}\frac{1}{\sqrt{2}}\big(\sqrt{3}\,|\vec{q}|M'V^0_{1/2\to1/2}+\sqrt{3}E'M'V^3_{1/2\to1/2}+M'^2V^1_{1/2\to3/2}-\sqrt{3}M'^2V^1_{1/2\to-1/2}\big) \tag{B.10}$$

$$C_6^V=\frac{1}{|\vec{q}|^3}\sqrt{\frac{E'+M'}{2M}}\frac{1}{\sqrt{2}}\left(-\sqrt{3}\,|\vec{q}|M'\frac{M-E'}{M}V^0_{1/2\to1/2}+\sqrt{3}\,|\vec{q}|^2\frac{M'}{M}V^3_{1/2\to1/2}\right) \tag{B.11}$$

[9] In sharp contrast, spin mixings however play a fundamental role in the case of the semileptonic [25,52] and electromagnetic [53] decays of the bc baryons.

$$C_3^A = -\frac{M'}{|\vec{q}|^2}\sqrt{\frac{E'+M'}{2M}}\frac{1}{\sqrt{2}}\left(A^1_{1/2\to 3/2} + \sqrt{3}A^1_{1/2\to -1/2}\right) \tag{B.12}$$

$$C_4^A = -\frac{M'}{|\vec{q}|}\frac{1}{\sqrt{(E'+M')2M}}\sqrt{\frac{3}{2}}\left(A^0_{1/2\to 1/2} + \frac{E'-M}{|\vec{q}|}A^3_{1/2\to 1/2}\right) + \frac{1}{M|\vec{q}|^2}\frac{1}{\sqrt{(E'+M')2M}}\frac{1}{\sqrt{2}}\left(\left(2M^2(E'+M') - MM'(M+M')\right)A^1_{1/2\to 3/2} + \sqrt{3}MM'(M+M')A^1_{1/2\to -1/2}\right) \tag{B.13}$$

$$C_5^A = \frac{M'}{|\vec{q}|}\frac{1}{\sqrt{(E'+M')2M}}\frac{ME'-M'^2}{M^2}\sqrt{\frac{3}{2}}\left(A^0_{1/2\to 1/2} + \frac{E'-M}{|\vec{q}|}A^3_{1/2\to 1/2}\right) + \frac{1}{M|\vec{q}|^2}\frac{1}{\sqrt{(E'+M')2M}}\frac{M'^2}{M}\left(2M(E'+M') - (M+M')^2\right)\frac{1}{\sqrt{2}}\left(A^1_{1/2\to 3/2} - \sqrt{3}A^1_{1/2\to -1/2}\right) \tag{B.14}$$

$$C_6^A = \frac{M'}{|\vec{q}|}\frac{1}{\sqrt{(E'+M')2M}}\sqrt{\frac{3}{2}}\left(A^0_{1/2\to 1/2} + \frac{E'}{|\vec{q}|}A^3_{1/2\to 1/2}\right) + \frac{M'^2}{|\vec{q}|^2}\frac{1}{\sqrt{(E'+M')2M}}\frac{1}{\sqrt{2}}\left(A^1_{1/2\to 3/2} - \sqrt{3}A^1_{1/2\to -1/2}\right) \tag{B.15}$$

For this latter case, 1/2-baryon to 3/2-baryon transitions, the following restrictions are observed

$$V^1_{1/2\to -1/2} = V^1_{-1/2\to 1/2}, \qquad V^1_{1/2\to 3/2} = \sqrt{3}V^1_{-1/2\to 1/2}, \qquad V^0_{1/2\to 1/2} = V^3_{1/2\to 1/2} = 0 \tag{B.16}$$

$$A^1_{1/2\to -1/2} = -A^1_{-1/2\to 1/2}, \qquad A^1_{1/2\to 3/2} = \sqrt{3}A^1_{-1/2\to 1/2} \tag{B.17}$$

so that

$$C_3^V = \frac{M'}{|\vec{q}|}\frac{1}{\sqrt{2M(E'+M')}}\sqrt{6}V^1_{-1/2\to 1/2} \tag{B.18}$$

$$C_4^V = -\frac{M}{M'}C_3^V \tag{B.19}$$

$$C_5^V = C_6^V = 0 \tag{B.20}$$

$$C_3^A = 0 \tag{B.21}$$

$$C_4^A = \frac{1}{\sqrt{(E'+M')2M}}\sqrt{\frac{3}{2}}\left[-\frac{M'}{|\vec{q}|}\left(A^0_{1/2\to 1/2} + \frac{E'-M}{|\vec{q}|}A^3_{1/2\to 1/2}\right) + \frac{2(ME'-M'^2)}{|\vec{q}|^2}A^1_{-1/2\to 1/2}\right] \tag{B.22}$$

$$C_5^A = \frac{M'}{|\vec{q}|}\frac{1}{\sqrt{(E'+M')2M}}\sqrt{\frac{3}{2}}\left[\frac{ME'-M'^2}{M^2}\left(A^0_{1/2\to 1/2} + \frac{E'-M}{|\vec{q}|}A^3_{1/2\to 1/2}\right) + \frac{2M'(2ME'-M^2-M'^2)}{M^2|\vec{q}|}A^1_{-1/2\to 1/2}\right] \tag{B.23}$$

$$C_6^A = \frac{M'}{|\vec{q}|}\frac{1}{\sqrt{(E'+M')2M}}\sqrt{\frac{3}{2}}\left(A^0_{1/2\to 1/2} + \frac{E'}{|\vec{q}|}A^3_{1/2\to 1/2} + \frac{2M'}{|\vec{q}|}A^1_{-1/2\to 1/2}\right) \tag{B.24}$$

The vector matrix elements have the general structure

$$V^0_{1/2\to 1/2} = V^{(0)}_{SF}\sqrt{2M}\sqrt{2E'}\int d^3Q_1\int d^3Q_2\left[\tilde{\phi}^{(B')}\left(\vec{Q}_1 - \frac{m_c+m_{l'}}{\bar{M}'}\vec{q}, -\vec{Q}_1 - \vec{Q}_2 + \frac{m_{l'}}{\bar{M}'}\vec{q}\right)\right]^*\tilde{\phi}^{(B)}(\vec{Q}_1,\vec{Q}_2) \times\sqrt{\frac{(E_l(|\vec{Q}_1-\vec{q}|)+m_l)(E_c(|\vec{Q}_1|)+m_c)}{2E_l(|\vec{Q}_1-\vec{q}|)2E_c(|\vec{Q}_1|)}}\left(1 + \frac{|\vec{Q}_1|^2 - |\vec{q}|Q_1^z}{(E_l(|\vec{Q}_1-\vec{q}|)+m_l)(E_c(|\vec{Q}_1|)+m_c)}\right) \tag{B.25}$$

$$V^3_{1/2\to 1/2} = V^{(3)}_{SF}\sqrt{2M}\sqrt{2E'}\int d^3Q_1\int d^3Q_2\left[\tilde{\phi}^{(B')}\left(\vec{Q}_1 - \frac{m_c+m_{l'}}{\bar{M}'}\vec{q}, -\vec{Q}_1 - \vec{Q}_2 + \frac{m_{l'}}{\bar{M}'}\vec{q}\right)\right]^*\tilde{\phi}^{(B)}(\vec{Q}_1,\vec{Q}_2) \times\sqrt{\frac{(E_l(|\vec{Q}_1-\vec{q}|)+m_l)(E_c(|\vec{Q}_1|)+m_c)}{2E_l(|\vec{Q}_1-\vec{q}|)2E_c(|\vec{Q}_1|)}}\left(\frac{Q_1^z}{E_c(|\vec{Q}_1|)+m_c} + \frac{Q_1^z - |\vec{q}|}{E_l(|\vec{Q}_1-\vec{q}|)+m_l}\right) \tag{B.26}$$

$$V^1_{-1/2\to 1/2} = V^{(1)}_{SF}\sqrt{2M}\sqrt{2E'}\int d^3Q_1\int d^3Q_2\left[\tilde{\phi}^{(B')}\left(\vec{Q}_1 - \frac{m_c+m_{l'}}{\bar{M}'}\vec{q}, -\vec{Q}_1 - \vec{Q}_2 + \frac{m_{l'}}{\bar{M}'}\vec{q}\right)\right]^*\tilde{\phi}^{(B)}(\vec{Q}_1,\vec{Q}_2) \times\sqrt{\frac{(E_l(|\vec{Q}_1-\vec{q}|)+m_l)(E_c(|\vec{Q}_1|)+m_c)}{2E_l(|\vec{Q}_1-\vec{q}|)2E_c(|\vec{Q}_1|)}}\frac{|\vec{q}|(E_c(|\vec{Q}_1|)+m_c) - [E_c(|\vec{Q}_1|)+m_c-E_l(|\vec{Q}_1-\vec{q}|)-m_l]Q_1^z}{(E_l(|\vec{Q}_1-\vec{q}|)+m_l)(E_c(|\vec{Q}_1|)+m_c)} \tag{B.27}$$

Here we have a $c\to l$ transition at the quark level, while l' is the light quark originally present in the initial baryon. The $V^{(j)}_{SF}$ depend on the flavour and spin structure of the baryons involved. Their values for the different transitions appear in Table 3. When the final baryon has just one s quark then $\tilde{\phi}^{(B')}$ should be interpreted as $\tilde{\phi}^{(B')}_{sn}$ or $\tilde{\phi}^{(B')}_{ds}$, for the case of $c\to s$ or $c\to d$ transitions, respectively.

Table 3
$V_{SF}^{(j)}$ and $A_{SF}^{(j)}$ spin-flavour factors.

	$V_{SF}^{(0)}$	$V_{SF}^{(3)}$	$V_{SF}^{(1)}$	$A_{SF}^{(0)}$	$A_{SF}^{(3)}$	$A_{SF}^{(1)}$
$\Xi_{cc}^{++} \to \Xi_c^+$	$\frac{\sqrt{3}}{\sqrt{2}}$	$\frac{\sqrt{3}}{\sqrt{2}}$	$\frac{-1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$
$\Xi_{cc}^{+} \to \Xi_c^0$	$\frac{\sqrt{3}}{\sqrt{2}}$	$\frac{\sqrt{3}}{\sqrt{2}}$	$\frac{-1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$
$\Xi_{cc}^{++} \to \Xi_c'^+$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{-5\sqrt{2}}{6}$	$\frac{5\sqrt{2}}{6}$	$\frac{5\sqrt{2}}{6}$	$\frac{5\sqrt{2}}{6}$
$\Xi_{cc}^{+} \to \Xi_c'^0$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{-5\sqrt{2}}{6}$	$\frac{5\sqrt{2}}{6}$	$\frac{5\sqrt{2}}{6}$	$\frac{5\sqrt{2}}{6}$
$\Xi_{cc}^{++} \to \Xi_c^{*+}$	0	0	$\frac{-1}{3}$	$\frac{-2}{3}$	$\frac{-2}{3}$	$\frac{1}{3}$
$\Xi_{cc}^{+} \to \Xi_c^{*0}$	0	0	$\frac{-1}{3}$	$\frac{-2}{3}$	$\frac{-2}{3}$	$\frac{1}{3}$
$\Xi_{cc}^{++} \to \Lambda_c^+$	$\frac{\sqrt{3}}{\sqrt{2}}$	$\frac{\sqrt{3}}{\sqrt{2}}$	$\frac{-1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$
$\Xi_{cc}^{++} \to \Sigma_c^+$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{-5\sqrt{2}}{6}$	$\frac{5\sqrt{2}}{6}$	$\frac{5\sqrt{2}}{6}$	$\frac{5\sqrt{2}}{6}$
$\Xi_{cc}^{+} \to \Sigma_c^0$	1	1	$\frac{-5}{3}$	$\frac{5}{3}$	$\frac{5}{3}$	$\frac{5}{3}$
$\Xi_{cc}^{++} \to \Sigma_c^{*+}$	0	0	$\frac{-1}{3}$	$\frac{-2}{3}$	$\frac{-2}{3}$	$\frac{1}{3}$
$\Xi_{cc}^{+} \to \Sigma_c^{*0}$	0	0	$\frac{-\sqrt{2}}{3}$	$\frac{-2\sqrt{2}}{3}$	$\frac{-2\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$
$\Omega_{cc}^{+} \to \Omega_c^0$	1	1	$\frac{-5}{3}$	$\frac{5}{3}$	$\frac{5}{3}$	$\frac{5}{3}$
$\Omega_{cc}^{+} \to \Omega_c^{*0}$	0	0	$\frac{-\sqrt{2}}{3}$	$\frac{-2\sqrt{2}}{3}$	$\frac{-2\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$
$\Omega_{cc}^{+} \to \Xi_c^0$	$\frac{-\sqrt{3}}{\sqrt{2}}$	$\frac{-\sqrt{3}}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	$\frac{-1}{\sqrt{6}}$	$\frac{-1}{\sqrt{6}}$	$\frac{-1}{\sqrt{6}}$
$\Omega_{cc}^{+} \to \Xi_c'^0$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{-5\sqrt{2}}{6}$	$\frac{5\sqrt{2}}{6}$	$\frac{5\sqrt{2}}{6}$	$\frac{5\sqrt{2}}{6}$
$\Omega_{cc}^{+} \to \Xi_c^{*0}$	0	0	$\frac{-1}{3}$	$\frac{-2}{3}$	$\frac{-2}{3}$	$\frac{1}{3}$

Similarly, for the axial matrix elements we have

$$A^0_{1/2\to1/2} = A^{(0)}_{SF}\sqrt{2M}\sqrt{2E'}\int d^3Q_1\int d^3Q_2\left[\tilde\phi^{(B')}\left(\vec Q_1 - \frac{m_c+m_{l'}}{\bar M'}\vec q, -\vec Q_1-\vec Q_2+\frac{m_{l'}}{\bar M'}\vec q\right)\right]^*\tilde\phi^{(B)}(\vec Q_1,\vec Q_2)$$
$$\times\sqrt{\frac{(E_l(|\vec Q_1-\vec q|)+m_l)(E_c(|\vec Q_1|)+m_c)}{2E_l(|\vec Q_1-\vec q|)2E_c(|\vec Q_1|)}}\left(\frac{Q_1^z}{E_c(|\vec Q_1|)+m_c}+\frac{Q_1^z-|\vec q|}{E_l(|\vec Q_1-\vec q|)+m_l}\right) \quad \text{(B.28)}$$

$$A^3_{1/2\to1/2} = A^{(3)}_{SF}\sqrt{2M}\sqrt{2E'}\int d^3Q_1\int d^3Q_2\left[\tilde\phi^{(B')}\left(\vec Q_1 - \frac{m_c+m_{l'}}{\bar M'}\vec q, -\vec Q_1-\vec Q_2+\frac{m_{l'}}{\bar M'}\vec q\right)\right]^*\tilde\phi^{(B)}(\vec Q_1,\vec Q_2)$$
$$\times\sqrt{\frac{(E_l(|\vec Q_1-\vec q|)+m_l)(E_c(|\vec Q_1|)+m_c)}{2E_n(|\vec Q_1-\vec q|)2E_c(|\vec Q_1|)}}\left(1-\frac{|\vec Q_1|^2-|\vec q|Q_1^z-2Q_1^z(Q_1^z-|\vec q|)}{(E_l(|\vec Q_1-\vec q|)+m_l)(E_c(|\vec Q_1|)+m_c)}\right) \quad \text{(B.29)}$$

$$A^1_{-1/2\to1/2} = A^{(1)}_{SF}\sqrt{2M}\sqrt{2E'}\int d^3Q_1\int d^3Q_2\left[\tilde\phi^{(B')}\left(\vec Q_1 - \frac{m_c+m_{l'}}{\bar M'}\vec q, -\vec Q_1-\vec Q_2+\frac{m_{l'}}{\bar M'}\vec q\right)\right]^*\tilde\phi^{(B)}(\vec Q_1,\vec Q_2)$$
$$\times\sqrt{\frac{(E_l(|\vec Q_1-\vec q|)+m_l)(E_c(|\vec Q_1|)+m_c)}{2E_l(|\vec Q_1-\vec q|)2E_c(|\vec Q_1|)}}\left(1-\frac{|\vec Q_1|^2-|\vec q|Q_1^z-2Q_1^x(Q_1^x-iQ_1^y)}{(E_l(|\vec Q_1-\vec q|)+m_l)(E_c(|\vec Q_1|)+m_c)}\right) \quad \text{(B.30)}$$

where the $A_{SF}^{(j)}$ axial spin-flavour factors can be found in Table 3. Note that due to symmetry properties the integral in $2Q_1^xQ_1^x$ in $A^1_{-1/2\to1/2}$ es equivalent to an integral in $|\vec Q_1|^2-(Q_1^z)^2$, while the integral in $2Q_1^xQ_1^y$ is identically zero.

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